Dynamic Programming and Applications Michael Schatz

Bioinformatics Lecture 2 Quantitative Biology 2013

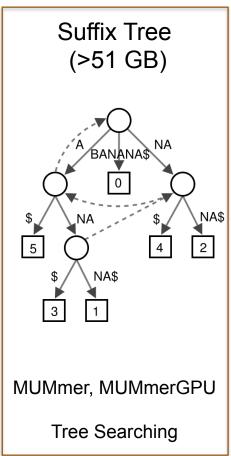


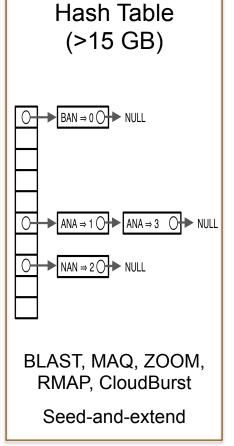
Exact Matching Review

Where is GATTACA in the human genome? E=183,105

Brute Force (3 GB) BANANA BAN ANA NAN ANA Naive Slow & Easy

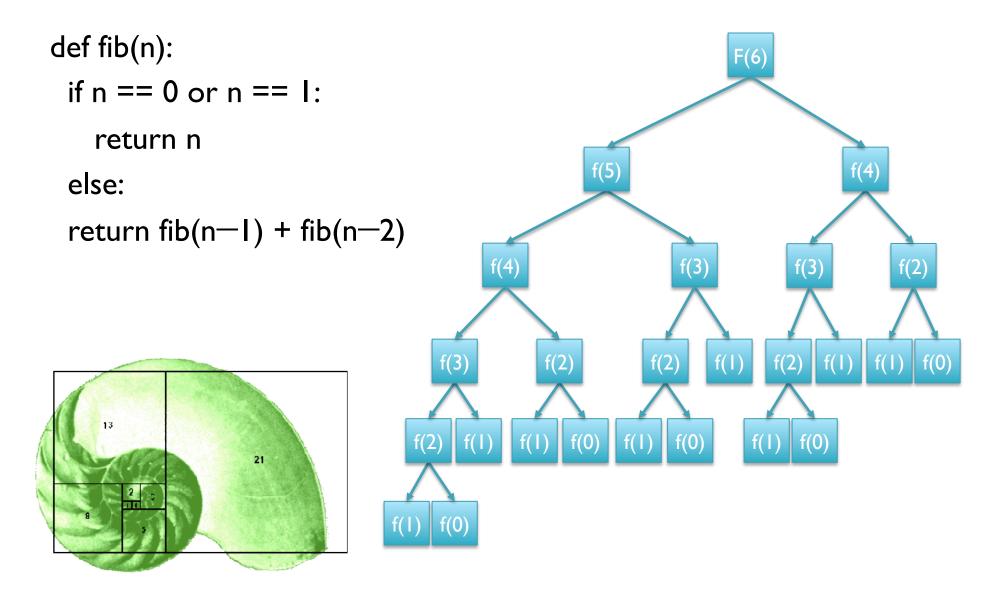
Suffix Array (>15 GB) A\$ ANA\$ ANANA\$ BANANA\$ NA\$ NANA\$ Vmatch, PacBio Aligner **Binary Search**



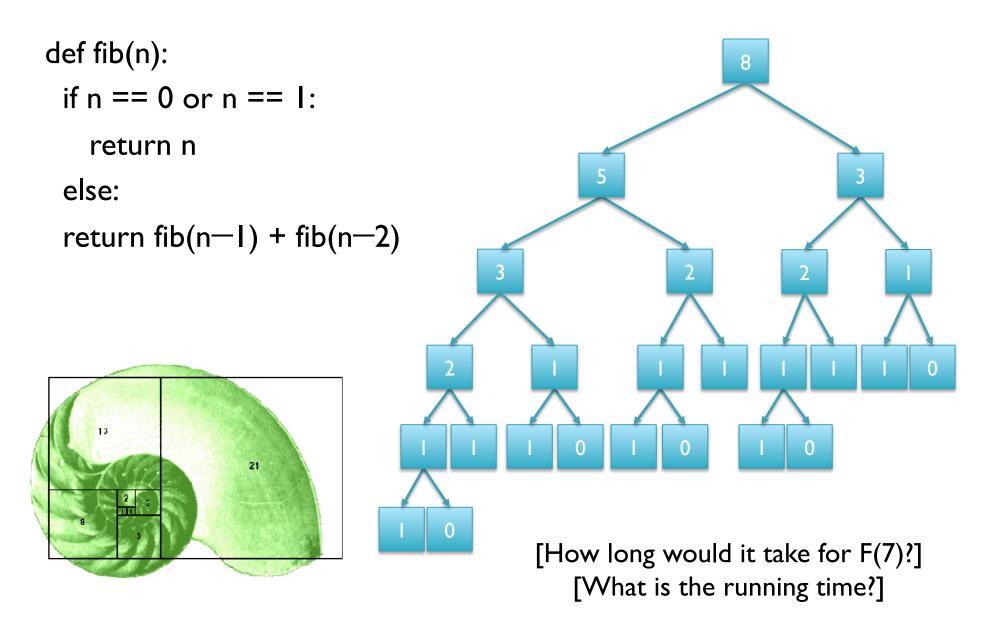


First: A quick warm-up exercise

Fibonacci Sequence

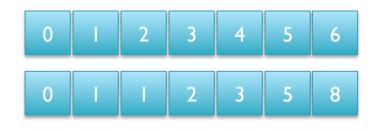


Fibonacci Sequence

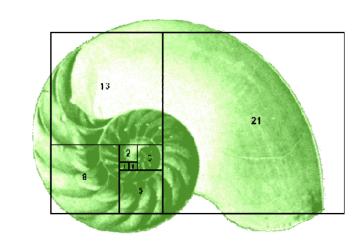


Bottom-up Fibonacci Sequence

```
def fib(n):
  table = [0] * (n+1)
  table[0] = 0
  table[1] = 1
  for i in range(2,n+1):
    table[i] = table[i-2] + table[i-1]
return table[n]
```

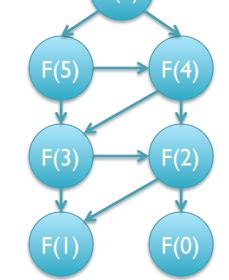


[How long will it take for F(7)?] [What is the running time?]



Dynamic Programming

- General approach for solving (some) complex problems
 - When applicable, the method takes far less time than naive methods.
 - Polynomial time $(O(n) \text{ or } O(n^2) \text{ instead of exponential time } (O(2^n) \text{ or } O(3^n))$
- Requirements:
 - Overlapping subproblems
 - Optimal substructure
- Applications:
 - Fibonacci
 - Longest Increasing Subsequence
 - Sequence alignment, Dynamic Time Warp, Viterbi



Not applicable:

- Traveling salesman problem, Clique finding, Subgraph isomorphism, ...
- The cheapest flight from airport A to airport B involves a single connection through airport C, but the cheapest flight from airport A to airport C involves a connection through some other airport D.

Longest Increasing Subsequence

- Given a sequence of N numbers $A_1, A_2, A_3, ..., A_N$, find the longest monotonically increasing subsequence
 - **29, 6, 14, 31, 39, 78, 63, 50, 13, 64, 61, 19**
- Greedy approach (always extend the subsequence if you can):
 - -29, 6, 14, 31, 39, 78, 63, 50, 13, 64, 61, 19 => 4
- Brute force:
 - Try all possible O(2ⁿ) subsequences

```
<u>29</u>, 6, 14, 31, 39, 78, 63, 50, 13, 64, 61, 19 => 1

<u>29</u>, <u>6</u>, 14, 31, 39, 78, 63, 50, 13, 64, 61, 19 => invalid

<u>29</u>, 6, <u>14</u>, 31, 39, 78, 63, 50, 13, 64, 61, 19 => invalid

<u>29</u>, 6, 14, <u>31</u>, 39, 78, 63, 50, 13, 64, 61, 19 => 2
```

. . .

Longest Increasing Subsequence

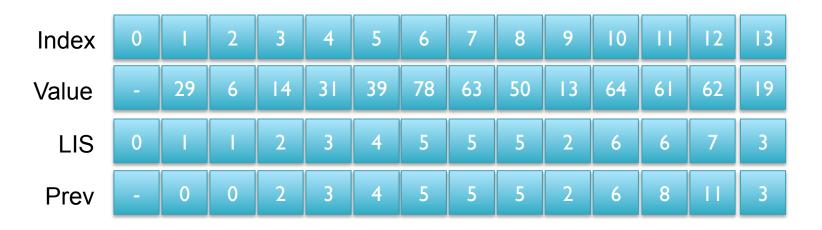
Idea:

- The solution for all N numbers depends on the solution for the first N-I
- Look through the previous values to find the longest subsequence ending at X such that $A_X < A_N$

Dynamic Programming:

Def: L[j] is the longest increasing subsequence ending at position j

- Base case: L[0] = 0 Recurrence: $L[j] = \max_{\substack{i < j \\ A[i] < A[j]}} \{L[i]\} + 1$ LIS= $\max\{L[i]\}$



[What's the LIS of 0,8,4,12,2,10,6,14,1,9,5,13,3,11,7,15?]

Longest Increasing Subsequence

```
// Initialize
L[0] = 0; P[0] = 0
// Iteratively apply recurrence
for i = I to N
    // find the best LIS to extend
    bestlis = 0; bestidx = -1
    for j = I to i
         if ((A[j] \le A[i])) && (L[j] > bestlis))
              bestlis = L[j]; bestidx = j
    L[i] = bestlis + I; P[i] = bestidx
// Scan the L array to find the overall LIS
LIS = 0
for j = I to N
    if (L[j] > LIS) LIS = L[j]
print "The LIS is $LIS"
```

```
      0
      I
      2
      3
      ...
      i
      ...
      j
      ...
      n

      -
      A[I]
      A[2]
      A[3]
      A[i]
      A[j]
      A[n]

      0
      L[I]
      L[2]
      L[3]
      L[i]
      L[i]+I

      0
      P[I]
      P[2]
      P[3]
      P[j]
      i
```

[What's the running time?]

In-exact alignment

- Where is GATTACA approximately in the human genome?
 - And how do we efficiently find them?
- It depends...
 - Define 'approximately'
 - Hamming Distance, Edit distance, or Sequence Similarity
 - Ungapped vs Gapped vs Affine Gaps
 - Global vs Local
 - All positions or the single 'best'?
 - Efficiency depends on the data characteristics & goals
 - Bowtie: BWT alignment for short read mapping
 - Smith-Waterman: Exhaustive search for optimal alignments
 - BLAST: Hash based homology searches
 - MUMmer: Suffix Tree based whole genome alignment

Similarity metrics

Hamming distance

Count the number of substitutions to transform one string into another

GATTACA	ATTACCC
	xx xx x
GATCACA	GATTACA
1	5

• Edit distance

 The minimum number of substitutions, insertions, or deletions to transform one string into another

GATTACA	-ATTACCC
	x xx
GATCACA	GATTAC-A
1	3

Edit Distance Example

AGCACACA → ACACACTA in 4 steps

```
AGCACACA \rightarrow (I. change G to C)
```

 $ACCACACA \rightarrow (2. delete C)$

ACACACA \rightarrow (3. change A to T)

ACACACT \rightarrow (4. insert A after T)

 $ACACACTA \rightarrow done$

[Is this the best we can do?]

Edit Distance Example

AGCACACA → ACACACTA in 3 steps

```
AGCACACA \rightarrow (1. change G to C)

ACCACACA \rightarrow (2. delete C)

ACACACA \rightarrow (3. insert T after 3<sup>rd</sup> C)

ACACACTA \rightarrow done
```

[Is this the best we can do?]

Reverse Engineering Edit Distance

D(AGCACACA, ACACACTA) = ?

Imagine we already have the optimal alignment of the strings, the last column can only be 1 of 3 options:

The optimal alignment of last two columns is then 1 of 9 possibilities

The optimal alignment of the last three columns is then 1 of 27 possibilities...

Eventually spell out every possible sequence of {I,M,D}

Reverse Engineering Edit Distance

D(AGCACACA, ACACACTA) = ?

Imagine we already have the optimal alignment of the strings, the last column can only be 1 of 3 options:

If these are the edit distances of the three possible optimal alignments up to the last column, the overall optimal alignment must use option M and the final distance is 2+0=2

This suggests an overall structure to the computation:

The edit distance of the overall alignment is computed from the edit distance of three possible alignments with the last column removed.... these values are themselves computed by removing the second to last column, and so forth down to the trivial base case of an empty string.

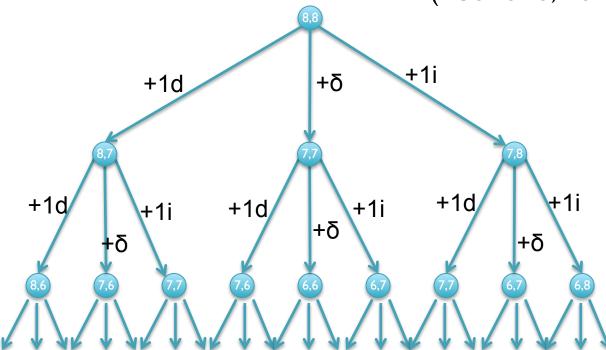
Recursive solution

- Computation of D is a recursive process.
 - At each step, we only allow matches, substitutions, and indels
 - D(i,j) in terms of D(i',j') for i' ≤ i and j' ≤ j.

```
D(AGCACACA, ACACACTA) = min{D(AGCACACA, ACACACT) + I,

D(AGCACAC, ACACACTA) + I,

D(AGCACAC, ACACACT) + \delta(A, A)}
```



[What is the running time?]

Dynamic Programming

- We could code this as a recursive function call...
 ...with an exponential number of function evaluations
- There are only (n+1)x(m+1) pairs i and j
 - We are evaluating D(i,j) multiple times
- Compute D(i,j) bottom up.
 - Start with smallest (i,j) = (1,1).
 - Store the intermediate results in a table.
 - Compute D(i,j) after D(i-1,j), D(i,j-1), and D(i-1,j-1)

Recurrence Relation for D

Find the edit distance (minimum number of operations to convert one string into another) in O(mn) time

Base conditions:

```
- D(i,0) = i, for all i = 0,...,n
- D(0,j) = j, for all j = 0,...,m
```

		A	С	A	С	A	С	Т	A
	0	I	2	3	4	5	6	7	8
A									
G	2								
С	3								
A	4								
С	5								
A	6								
С	7								
A	8								

[What does the initialization mean?]

		A	С	A	С	A	С	Т	Α
	0		2	3	4	5	6	7	8
A	_	0							
G	2								
С	3								
A	4								
С	5								
A	6								
С	7								
Α	8								

 $D[A,A] = min\{D[A,]+1, D[,A]+1, D[,]+\delta(A,A)\}$

		A	С	A	С	A	С	Т	A
	0		2	3	4	5	6	7	8
A		0	1						
G	2								
С	3								
A	4								
С	5								
A	6								
С	7								
A	8								

 $D[A,AC] = min\{D[A,A]+1, D[,AC]+1, D[,A]+\delta(A,C)\}$

		A	С	A	С	A	С	Т	A
	0	I	2	3	4	5	6	7	8
A		0	_	2					
G	2								
С	3								
A	4								
С	5								
A	6								
С	7								
A	8								

 $D[A,ACA] = min\{D[A,AC]+1, D[,ACA]+1, D[,AC]+\delta(A,A)\}$

		A	С	A	С	A	С	Т	A
	<u>0</u>	<u></u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	8
A	I	0	I	2	3	4	5	6	<u>7</u>
G	2								
С	3								
A	4								
С	5								
A	6								
С	7								
A	8								

[What about the other A?]

		A	С	A	С	A	С	Т	A
	<u>0</u>	<u></u>	<u>2</u>	<u>3</u>	<u>4</u>	5	6	7	8
A		0		2	3	<u>4</u>	5	6	7
G	2	I	I	2	3	4	<u>5</u>	<u>6</u>	<u>7</u>
С	3								
A	4								
С	5								
A	6								
С	7								
A	8								

		A	U	A	U	A	С	Т	Α
	0	_	2	3	4	5	6	7	8
A	_	0		2	3	4	5	6	7
G	2	<u> </u>		2	3	4	5	6	7
С	3	2		2	2	3	4	5	6
A	4	3	2	<u> </u>	2	2	3	4	5
С	5	4	3	2	—	2	2	3	4
A	6	5	4	3	2	<u></u>	2	3	3
С	7	6	5	4	3	2	_	<u>2</u>	3
A	8	7	6	5	4	3	2	2	<u>2</u>

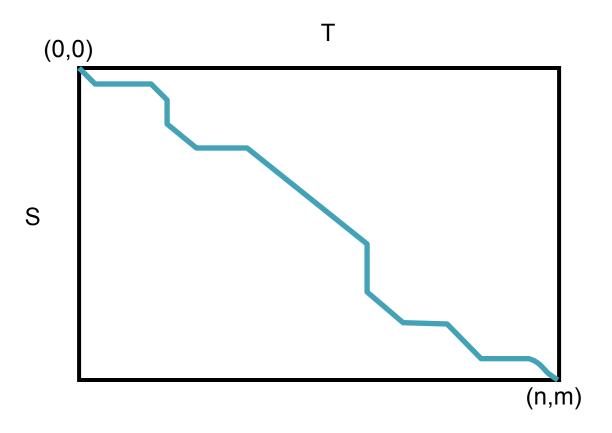
[Can we do it any better?]

Break

		A	U	A	U	A	С	Т	Α
	0	_	2	3	4	5	6	7	8
A	_	0		2	3	4	5	6	7
G	2	<u> </u>		2	3	4	5	6	7
С	3	2		2	2	3	4	5	6
A	4	3	2	<u> </u>	2	2	3	4	5
С	5	4	3	2	—	2	2	3	4
A	6	5	4	3	2	<u></u>	2	3	3
С	7	6	5	4	3	2	_	<u>2</u>	3
A	8	7	6	5	4	3	2	2	<u>2</u>

[Can we do it any better?]

Global Alignment Schematic



- A high quality alignment will stay close to the diagonal
 - If we are only interested in high quality alignments, we can skip filling in cells that can't possibly lead to a high quality alignment
 - Find the global alignment with at most edit distance d: O(2dn)

Sequence Similarity

- Similarity score generalizes edit distance
 - Certain mutations are much more likely than others
 - Hydrophilic -> Hydrophillic much more likely than Hydrophillic -> Hydrophobic
 - BLOSSUM62
 - Empirically measure substitution rates among proteins that are 62% identical
 - Positive score: more likely than chance, Negative score: less likely

Edit Distance and Global Similarity

```
D(i,j) = min \{
                D(i-1,j) + 1,
                D(i,j-1)+1,
                D(i-1,j-1) + \delta(S(i),T(j))
s = 4x4 or 20x20 scoring matrix
S(i,j) = max {
                S(i-1,j) + 1,
                                                         [Why max?]
                S(i,j-1) + I
                S(i-1,j-1) + s(S(i),T(j))
```

Local vs. Global Alignment

• The Global Alignment Problem tries to find the best path between vertices (0,0) and (n,m) in the edit graph.

• The Local Alignment Problem tries to find the best path among paths between **arbitrary vertices** (*i,j*) and (*i'*, *j'*) in the edit graph.

[How many (i,j) x (i',j') pairs are there?]

Local vs. Global Alignment (cont'd)

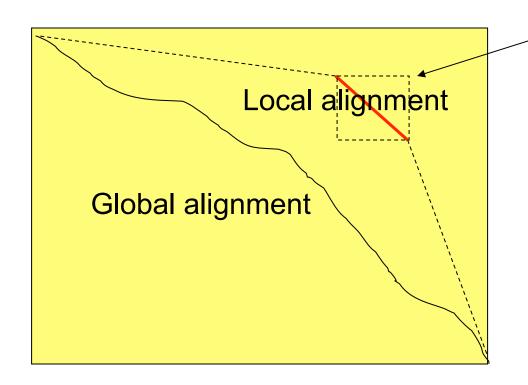
Global Alignment

Local Alignment—better alignment to find conserved segment

tccCAGTTATGTCAGgggacacgagcatgcagagac

aattgccgccgtcgttttcagCAGTTATGTCAGatc

Local Alignment: Example



Compute a "mini"
Global Alignment to
get Local

The Local Alignment Recurrence

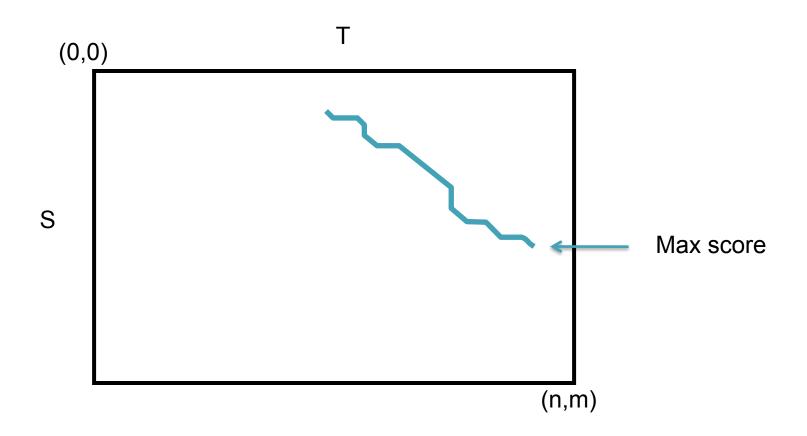
• The largest value of $s_{i,j}$ over the whole edit graph is the score of the best local alignment.

The recurrence:

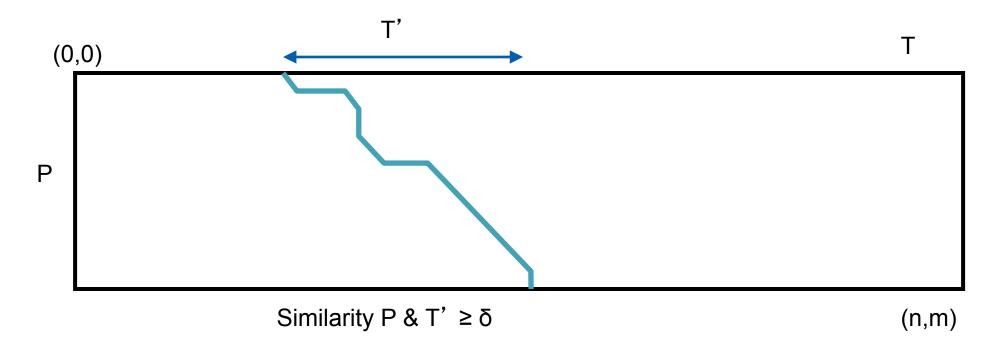
$$S_{i,j} = max \begin{cases} 0 \\ S_{i-1,j-1} + \delta(v_i, w_j) \\ S_{i-1,j} + \delta(v_i, -) \\ S_{i,j-1} + \delta(-, w_j) \end{cases}$$

Power of ZERO: there is only this change from the original recurrence of a Global Alignment - since there is only one "free ride" edge entering into every vertex

Local Alignment Schematic



Searching for GATTACA

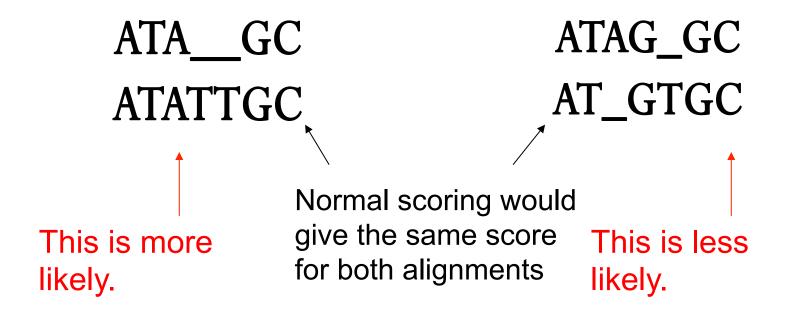


- Don't "charge" for optimal alignment starting in cells (0,j)
 - Base conds: D(0,j) = 0, $D(i,0) = \Sigma_{k \le i} s(S(k), '-')$
- Don't "charge" for ending alignment at end of P (but not necc.T)
 - Find cell (n,j) with edit distance $\leq \delta$

Nathan Edwards

Affine Gap Penalties

• In nature, a series of k indels often come as a single event rather than a series of k single nucleotide events:



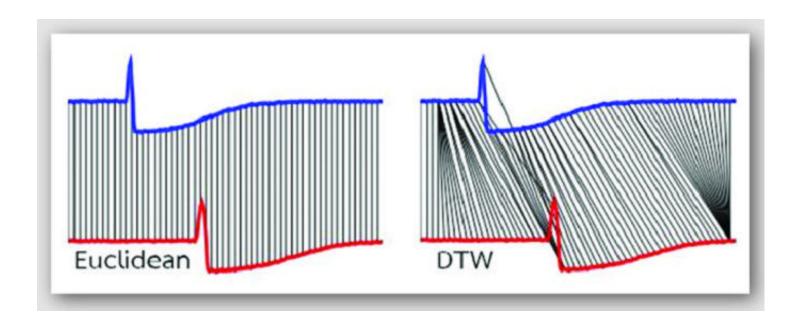
Accounting for Gaps

- Gaps- contiguous sequence of spaces in one of the rows
- Score for a gap of length x is: $-(\rho + \sigma x)$ where $\rho > 0$ is the gap opening penalty ρ will be large relative to gap extension penalty σ
 - Gap of length $1:-(\rho + \sigma) = -6$
 - Gap of length 2: $-(\rho + \sigma 2) = -7$
 - Gap of length 3: $-(\rho + \sigma 3) = -8$

 Smith-Waterman-Gotoh incorporates affine gap penalties without increasing the running time O(mn)

Dynamic Time Warp

- Algorithm for measuring the similarity between two sequences of numeric values that vary in time or speed
 - Computes a non-linear mapping for sequence A to sequence B
 - Many applications for video, audio, and graphics
 - Speech processing: Recognize speech patterns coping with different speaking speeds
 - EEG processing: Identify anomalies in brain or heart activity



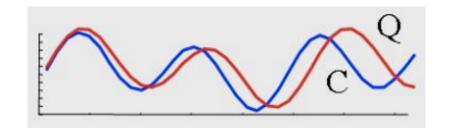
DTW Algorithm

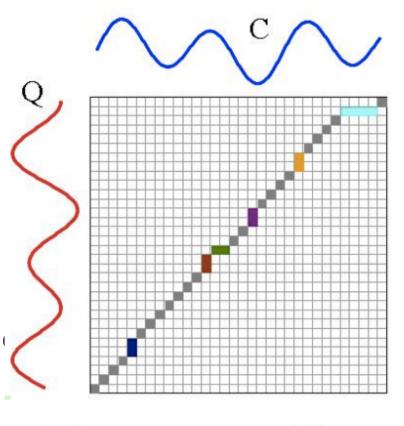
- DP Algorithm
 - Input: two time series C and Q
 - Compute the time warping matrix d

$$d(0,0) = 0; d(i,0) = d(0,j) = \infty$$

$$d(i,j) = |c_i - q_j| + \min \begin{cases} d(i-1, j) \\ d(i,j-1) \\ d(i-1, j-1) \end{cases}$$

 Warping matrix projects sequence to sequence Q, allowing for nonlinear contractions and expansions.





Break

Basic Local Alignment Search Tool

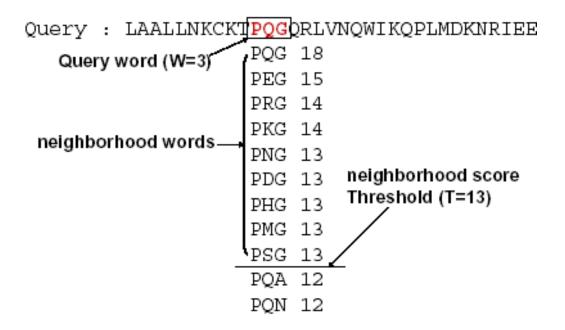
- Rapidly compare a sequence Q to a database to find all sequences in the database with an score above some cutoff S.
 - Which protein is most similar to a newly sequenced one?
 - Where does this sequence of DNA originate?
- Speed achieved by using a procedure that typically finds "most" matches with scores > S.
 - Tradeoff between sensitivity and specificity/speed
 - Sensitivity ability to find all related sequences
 - Specificity ability to reject unrelated sequences

Seed and Extend

FAKDFLAGGVAAAISKTAVAPIERVKLLLQVQHASKQITADKQYKGIIDCVVRIPKEQGV F D +GG AAA+SKTAVAPIERVKLLLQVQ ASK I DK+YKGI+D ++R+PKEQGV FLIDLASGGTAAAVSKTAVAPIERVKLLLQVQDASKAIAVDKRYKGIMDVLIRVPKEQGV

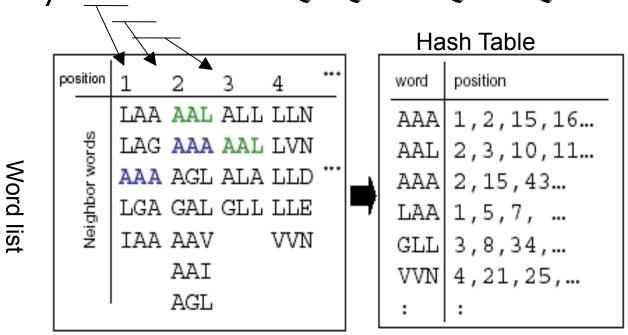
- Homologous sequence are likely to contain a short high scoring word pair, a seed.
 - Unlike Baeza-Yates, BLAST *doesn't* make explicit guarantees
- BLAST then tries to extend high scoring word pairs to compute maximal high scoring segment pairs (HSPs).
 - Heuristic algorithm but evaluates the result statistically.

Step I: Preprocess Query
 Compile the short-high scoring word list from query.
 The length of query word, w, is 3 for protein scoring
 Threshold T is 13

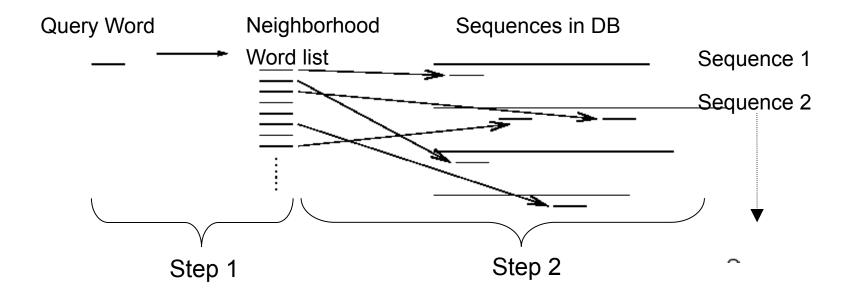


• Step 2: Construct Query Word Hash Table

Query: LAALLNKCKTPQGQRLVNQWIKQPLMD



Step 3: Scanning DB
 Identify all exact matches with DB sequences



- Step 4 (Search optimal alignment)
 For each hit-word, extend ungapped alignments in both directions.
 Let S be a score of hit-word
- Step 5 (Evaluate the alignment statistically)
 Stop extension when E-value (depending on score S) become less than threshold. The extended match is called High Scoring Segment Pair.

E-value = the number of HSPs having score S (or higher) expected to occur by chance.

→ Smaller E-value, more significant in statistics
Bigger E-value, by chance

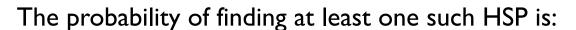
E[# occurrences of a string of length m in reference of length L] $\sim L/4^{m}$

BLAST E-values

The expected number of HSPs with the score at least S is:

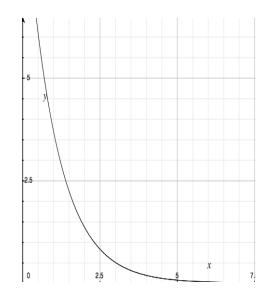
$$E = K*n*m*e^{-\lambda S}$$

K, λ are constant depending on model n, m are the length of query and sequence



$$P = I - e^{E}$$

→ If a word is hit by chance (E-value is bigger),
P become smaller.



The distribution of Smith-Waterman local alignment scores between two random sequences follows the Gumbel extreme value distribution

Parameters

- Larger values of w increases the number of neighborhood words, but decreases the number of chance matches in the database.
 - Increasing w decreases sensitivity.
- Larger values of T decrease the overall execution time, but increase the chance of missing a MSP having score ≥ S.
 - Increases T decreases the sensitivity
- Larger values of S increase the specificity. The value of S is affected by changes in the expectation value parameter.

Very Similar Sequences

```
Query: HBA HUMAN Hemoglobin alpha subunit
Sbjct: HBB HUMAN Hemoglobin beta subunit
Score = 114 bits (285), Expect = 1e-26
Identities = 61/145 (42%), Positives = 86/145 (59%), Gaps = 8/145 (5%)
Query 2 LSPADKTNVKAAWGKVGAHAGEYGAEALERMFLSFPTTKTYFPHF-----DLSHGSAQV 55
          L+P+K+V A WGKV + E G EAL R+ + +P T+ +F F
                                                                G+ +V
Sbjct 3 LTPEEKSAVTALWGKV--NVDEVGGEALGRLLVVYPWTQRFFESFGDLSTPDAVMGNPKV 60
Ouery 56 KGHGKKVADALTNAVAHVDDMPNALSALSDLHAHKLRVDPVNFKLLSHCLLVTLAAHLPA 115
                                   + LS+LH KL VDP NF+LL + L+ LA H
          K HGKKV A ++ +AH+D++
Sbjct
      61 KAHGKKVLGAFSDGLAHLDNLKGTFATLSELHCDKLHVDPENFRLLGNVLVCVLAHHFGK 120
Query 116 EFTPAVHASLDKFLASVSTVLTSKY 140
          EFTP V A+ K +A V+ L KY
Sbjct 121 EFTPPVOAAYOKVVAGVANALAHKY 145
```

Quite Similar Sequences

```
Query: HBA HUMAN Hemoglobin alpha subunit
Sbjct: MYG HUMAN Myoglobin
Score = 51.2 bits (121), Expect = 1e-07,
Identities = 38/146 (26%), Positives = 58/146 (39%), Gaps = 6/146 (4%)
Ouerv 2 LSPADKTNVKAAWGKVGAHAGEYGAEALERMFLSFPTTKTYFPHF-----DLSHGSAOV
                                                                       55
         LS +
                 V
                     WGKV A
                               +G E L R+F
                                            PT
Sbjct 3 LSDGEWQLVLNVWGKVEADIPGHGQEVLIRLFKGHPETLEKFDKFKHLKSEDEMKASEDL
                                                                       62
Ouerv 56 KGHGKKVADALTNAVAHVDDMPNALSALSDLHAHKLRVDPVNFKLLSHCLLVTLAAHLPA
                                                                       115
         K HG V AL
                                 + L+ HA K ++
                                                    + +S C++ L + P
Sbjct
      63 KKHGATVLTALGGILKKKGHHEAEIKPLAQSHATKHKIPVKYLEFISECIIQVLQSKHPG
                                                                       122
Query 116 EFTPAVHASLDKFLASVSTVLTSKYR 141
           +F
                  +++K L
                              + S Y+
Sbjct 123 DFGADAOGAMNKALELFRKDMASNYK
                                      148
```

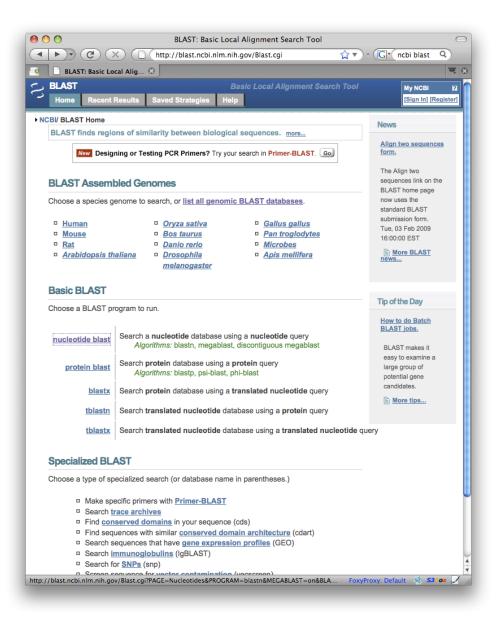
Not similar sequences

```
Query: HBA HUMAN Hemoglobin alpha subunit
Sbjct: SPAC869.02c [Schizosaccharomyces pombe]
 Score = 33.1 bits (74), Expect = 0.24
 Identities = 27/95 (28%), Positives = 50/95 (52%), Gaps = 10/95 (10%)
Query
      30 ERMFLSFPTTKTYFPHFDLSHGSAOVKGHGKKVADALTNAVAHVDDMPNALSALSDLHAH 89
          ++M ++P
                        P+F+ +H +
                                        + +A AL N
                                                   ++DD+
                                                          +LSA D
Sbjct
      59 QKMLGNYPEV---LPYFNKAHQISL--SQPRILAFALLNYAKNIDDL-TSLSAFMDQIVV 112
Ouery 90 K---LRVDPVNFKLLSHCLLVTLAAHLPAEF-TPA
                                              120
              L++
                    ++ ++ HCLL T+
                                    LP++ TPA
      113 KHVGLQIKAEHYPIVGHCLLSTMQELLPSDVATPA 147
```

Blast Versions

Program	Database	Query
BLASTN	Nucleotide	Nucleotide
BLASTP	Protein	Protein
BLASTX	Protein	Nucleotide translated in to protein
TBLASTN	Nucleotide translated in to protein	Protein
TBLASTX	Nucleotide translated in to protein	Nucleotide translated in to protein

NCBI Blast



Nucleotide Databases

- nr:All Genbank
- refseq: Reference organisms
- wgs:All reads

Protein Databases

- nr:All non-redundant sequences
- Refseq: Reference proteins

BLAST Exercise

>whoami

TTGATGCAGGTATCTGCGACTGAGACAATATGCA
ACAGTTGAATGAATCATAATGGAATGTGCACTCT
AACCAGCCAATTTGATGCTGGCTGCAGAGATGC
AAGATCAAGAGGTGACACCTGCTCTGAAGAAAG
CACAGTTGAACTGCTGGATCTGCAACTACAGCA
GGCACTCCAGGCACCAAGACAACATCTTTTACA
CCAGCAAACATGTGGATTGATATCTCCTAACAGC
AGTGATTAACAGAGACGACTGCAGGATTTGCTTC
CACAAACAAAAT

Sequence Alignment Summary

Distance metrics:

- Hamming: How many substitutions?
- Edit Distance: How many substitutions or indels?
- Sequence Similarity: How similar (under this model of similarity)?

Techniques

- Seed-and-extend: Anchor the search for in-exact using exact only
- Dynamic Programming: Find a global optimal as a function of its parts
- BWT Search: implicit DFS of SA/ST

Sequence Alignment Algorithms: Pick the right tool for the job

- Smith-Waterman: DP Local sequence alignment
- BLAST: Homology Searching
- MUMmer: Whole genome alignment
- Bowtie/BWA/Novoalign: short read mapping

Thank You!

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